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Stiff Domain Walls in Creation-Field Cosmology

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Abstract Hoyle-Narlikar C-field has been introduced to study domain walls in Bianchi type-I space-time. The solutions have been studied when the creation field C is a function of time t only. The geometrical and physical aspects for model have also been studied.

Keywords Bianchi type-I universe · Creation field cosmology · Cosmological model of universe · Stiff domain walls · Deceleration parameter

1 Introduction

Topological defects associated with spontaneous symmetry breaking are extremely interesting and attractive features of gauge theories [1]. A large amount of the structure of unknown Universe may have resulted from the formation of different types of topological defects [2]. It is assumed that at very early stages of its evolution, the Universe has gone through a number of phase transitions and of which could have resulted in the formation of topological defects, which may be all quite different in character. These include point like defect known as monopoles, string like defect known as cosmic strings and domain walls which are sheet like defects [3]. Hill, Schramm and Fry [4] have suggested that light domain walls of large thickness may have been produced during the late time phase transition. Vilenkin [5] first showed that the gravitation field of an infinite thin domain wall with planar symmetry cannot be described by a static metric. Subsequently, Widrow [6] noted that nor could a thick domain wall can be described by a regular static metric. These considerations suggest that non-static metrics are suitable for description of the field of thick domain wall. Many authors have discussed non-static solutions of the Einstein scalar field equations for thick domain walls [6–8]. But these solutions have peculiar behaviour. In these solutions the energy scalar is independent of time whereas the metric tensor depends on both space and time. Letelier and Wang [9] have obtained exact solutions to the Einstein field equations that represent the

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collision of thin domain walls. Thick domain walls are characterized by the energy momentum tensor

$$^{m}T_{ij} = \rho(g_{ij} + w_i w_j) + p w_i w_j, \qquad w^i w_j = -1, \tag{1.1}$$

where ρ is the rest energy density of the wall, p is the pressure in the direction normal to the plane of the wall and w_i is the unit space like vector in the same direction.

There are two approaches for the study of thick domain walls. In the first approach one studies the field equations as well as the equation of domain wall treated as the self interacting scalar field. In the second approach one assumes the energy momentum tensor in the form (1.1) and then the field equations are solved. In the present paper we adopt the second approach. Isper and Sikivie [10], Rahmann et al. [11], Reddy and Subbarao [12], Adhav et al. [13], Patel et al. [14] are some of the authors who have investigated several aspects of domain walls.

Many authors have studied the solutions of the Einstein's field equations (EFEs) for homogenous and Bianchi type models, in recent years e.g. Haji-Boutros [15, 16], ShriRam [17, 18], Mazumder [19], Camci et al. [20] and Pradhan and Kumar [21] using different generating techniques. Solutions of the field equations may also be generated by applying a law of variation of Hubble parameter, which was initially proposed by Berman [22], for FRW models. The law yields a constant value of the deceleration parameter. The theory of the constant deceleration parameter has been further developed by Berman and Gomide [23]. It should be remarked that the formula is independent of the particular gravitational theory being considered it is a property valid for FRW metric, and it is approximately valid also for slowly time varying deceleration parameter. In literature, cosmological model with the constant deceleration parameter have been studied by Johri and Desikan [24], Singh and Desikan [25], Maharaj and Naidoo [26], Pradhan et al. [27], Pradhan and Vishwakarma [28, 29], Rahaman et al. [30], Reddy et al. [31], and others in different theories of FRW and Bianchi type I models. Recently series of work, Singh and Kumar [32] and Kumar and Singh [33] extended Berman's work for the anisotropic Bianchi type I and II space-time models by formulating a law of variation for the mean Hubble parameter and found the solutions to EFEs in the simplest way. Akarsu and Kilimc [34] studied locally rotationally symmetric Bianchi type-I cosmological model in the presence of dark energy and perfect fluid using the law of variation for the mean Hubble parameter.

The three important observations in astronomy viz., the phenomenon of expanding universe, primordial nucleon-synthesis and the observed isotropy of cosmic microwave background radiation (CMBR) were supposed to be successfully explained by big-bang cosmology based on Einstein's field equations. However, Smoot et al. [35] revealed that the earlier predictions of the Friedman-Robertson-Walker type of models do not always exactly meet our expectations. Some puzzling results regarding the red shifts from the extra galactic objects continue to contradict the theoretical explanations given from the big bang type of the model. Also, CMBR discovery did not prove it to be a out come of big bang theory. In fact, Narlikar et al. [36] have proved the possibility of non-relic interpretation of CMBR. To explain such phenomenon, many alternative theories have been proposed from time to time. Hoyle [37], Bondi and Gold [38] proposed steady state theory in which the universe does not have singular beginning nor an end on the cosmic time scale. Moreover, they have shown that the statistical properties of the large scale features of the universe do not change. Further, the constancy of the mass density has been accounted by continuous creation of matter going on in contrast to the one time infinite and explosive creation of matter at $t = 0$ as in the earlier standard model. But the principle of conservation of matter was violated in this formalism. To overcome this difficulty Hoyle and Narlikar [39] adopted a field theoretic approach by introducing a massless and chargeless scalar field C in the Einstein-Hilbert action to account for the matter creation. In the C-field theory introduced by Hoyle and Narlikar there is no big bang type of singularity as in the steady state theory of Bondi and Gold [38]. A solution of Einstein's field equations admitting radiation with negative energy massless scalar creation fields C was obtained by Narlikar and Padmanabhan [40]. The study of Hoyle and Narlikar theory [39, 41, 42] to the space-time of dimensions more than four was carried out by Chatterjee and Banerjee [43]. RajBali and Tikekar [44] studied C-field cosmology with variable G in the flat Friedmann-Robertson-Walker model. C-field cosmological models with variable G in FRW space-time have been studied by RajBali and Kumawat [45]. The solutions of Einstein's field equations in the presence of creation field have been obtained for axially symmetric universe in four dimensions by Singh and Chaubey [46].

We have obtained exact solutions for Bianchi type-I metric by assuming a special law of the mean Hubble parameter which yields a constant deceleration parameter. The assumption on the mean Hubble parameter allows us to determine the scale factors exactly as well as other cosmological parameters of such a universe.

2 Hoyle and Narlikar C-field Cosmology

Introducing a massless scalar field called as creation field viz. C-field, the Einstein's field equations (Hoyle and Narlikar [39, 41, 42]) are modified.

The modified Einstein's field equations are ($8\pi G = 1$)

$$R_{ij} - \frac{1}{2} g_{ij} R = -({}^m T_{ij} + {}^c T_{ij}), \tag{2.1}$$

where ${}^m T_{ij}$ is matter tensor of Einstein theory and ${}^c T_{ij}$ is matter tensor due to the C-field which is given by

$${}^c T_{ij} = -f\left(C_i C_j - \frac{1}{2} g_{ij} C^k C_k\right), \tag{2.2}$$

where $f > 0$ and $C_i = \frac{\partial C}{\partial x^i}$.

Because of the negative value of T^{00} ($T^{00} < 0$), the C-field has negative energy density producing repulsive gravitational field which causes the expansion of the universe. Hence, the energy conservation equation reduces to

$${}^m T^{ij}_{;j} = -{}^c T^{ij}_{;j} = f C^i C^j_{;j}, \tag{2.3}$$

i.e. matter creation through non-zero left hand side is possible while conserving the over all energy and momentum.

Above equation is similar to

$$m g_{ij} \frac{dx^i}{ds} - C_j = 0, \tag{2.4}$$

which implies that the 4-momentum of the created particle is compensated by the 4-momentum of the C-field. In order to maintain the balance, the C-field must have negative energy.

3 Metric and Field Equations

The Bianchi-Type-I line element can be written as

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 dy^2 - a_3^2 dz^2, \tag{3.1}$$

where a_1, a_2, and a_3 are functions of t only.

We have assumed that creation field C is function of time t only i.e.

$$C(x,t) = C(t). \tag{3.2}$$

We have also assumed that velocity of light is equal to one unit.

Now, the field equations (2.1) for metric (3.1) with the help of (1.1) and (2.2) yield

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} = \rho - \frac{1}{2}f\dot{C}^2, \tag{3.3}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} = -p + \frac{1}{2}f\dot{C}^2, \tag{3.4}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} = \rho + \frac{1}{2}f\dot{C}^2, \tag{3.5}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} = \rho + \frac{1}{2}f\dot{C}^2, \tag{3.6}$$

where dot $(\cdot)$ indicates the derivative with respect to t.

The field equations (3.3) to (3.6) are four field equations in six unknowns a_1, a_2, a_3, ρ, p and C. Hence to obtain a determinate solution we have to assume some physical or mathematical condition. So we impose a law of variation for the Hubble parameter to get constant deceleration parameter.

According to this law the variation of the mean Hubble parameter H for Bianchi type-I metric is given by

$$H = k(a_1a_2a_3)^{-m/3}, \tag{3.7}$$

where $k > 0$ and $m \geq 0$ are constants.

The spatial volume is given by

$$V = a^3 = \prod_{i=1}^{3} a_i, \tag{3.8}$$

where a is the mean scale factor.

The mean Hubble parameter H for Bianchi type-I metric is also given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right). \tag{3.9}$$

The directional Hubble parameters in the directions of x-, y- and z-axis respectively are defined as

$$H_x = \frac{\dot{a}_1}{a_1}, \qquad H_y = \frac{\dot{a}_2}{a_2}, \qquad H_z = \frac{\dot{a}_3}{a_3}. \tag{3.10}$$

The volumetric deceleration parameter q is given by

$$q = -\frac{a\ddot{a}}{\dot{a}^2}. \tag{3.11}$$

Equating (3.7) and (3.9) and then integrating we get

$$V = a_1 a_2 a_3 = c_1 e^{3kt} \quad \text{for } m = 0, \tag{3.12}$$

and

$$V = a_1 a_2 a_3 = (mkt + c_2)^{\frac{3}{m}} \quad \text{for } m \neq 0, \tag{3.13}$$

where c_1 and c_2 are positive constants of integration.

Using (3.7) for $m = 0$ with (3.12) and (3.7) for $m \neq 0$ with (3.13), the mean Hubble parameter becomes

$$H = k \quad \text{for } m = 0, \tag{3.14}$$

$$H = k(mkt + c_2)^{-1} \quad \text{for } m \neq 0. \tag{3.15}$$

Using (3.12), (3.13) and (3.8) in (3.11) we get, constant values for the deceleration parameter for mean scale factor as:

$$q = -1 \quad \text{for } m = 0, \tag{3.16}$$

$$q = m - 1 \quad \text{for } m \neq 0, \tag{3.17}$$

The sign of q indicates whether the model accelerates or not. The positive sign if q $(m > 1)$ corresponds to decelerating models whereas the negative sign $-1 \leq q < 0$ for $0 \leq m < 1$ indicates acceleration and $q = 0$ for $m = 1$ corresponds to expansion with constant velocity.

Subtracting (3.5) from (3.6), we get

$$\frac{d}{dt}\left(\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3}\right) + \left(\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3}\right)\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) = 0. \tag{3.18}$$

Now, from (3.9) and (3.18), we get

$$\frac{d}{dt}\left(\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3}\right) + \left(\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3}\right)(3H) = 0.$$

Integrating, which gives

$$\left(\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3}\right) = \beta_1 e^{-3kt}, \quad \text{for } m = 0, \tag{3.19}$$

$$\left(\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3}\right) = \frac{\beta_1}{(mkt + c_2)^{\frac{3}{m}}}, \quad \text{for } m \neq 0 \text{ and } 3, \tag{3.20}$$

and

$$\left(\frac{\dot{a}_2}{a_2} - \frac{\dot{a}_3}{a_3}\right) = \frac{\beta_1}{(3kt + c_2)} \quad \text{for } m = 3, \tag{3.21}$$

where $\beta_1 =$ constant of integration.

Adding (3.4), (3.5), (3.6) and 3 times (3.3), we get

$$\left(\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}\right)+2\left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right)=\frac{5\rho-p}{2} \tag{3.22}$$

From (3.8) we have

$$\frac{\ddot{V}}{V}=\left(\frac{\ddot{a}_1}{a_1}+\frac{\ddot{a}_2}{a_2}+\frac{\ddot{a}_3}{a_3}\right)+2\left(\frac{\dot{a}_1\dot{a}_2}{a_1a_2}+\frac{\dot{a}_2\dot{a}_3}{a_2a_3}+\frac{\dot{a}_3\dot{a}_1}{a_3a_1}\right) \tag{3.23}$$

From (3.22) and (3.23), we get

$$\frac{\ddot{V}}{V}=\frac{5\rho-p}{2} \tag{3.24}$$

The relation between ρ and p, is given by the proportionality relation:

$$\rho=\gamma p \tag{3.25}$$

where γ being the proportionality constant. To get a deterministic solution we consider the case stiff domain walls ($\rho = p$).

The physical parameters which are of observational importance are expansion scalar θ, the mean anisotropy parameter Δ and the shear scalar σ^2. These parameters can respectively be obtained by using the formulas

$$\theta=3H, \tag{3.26}$$

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2, \tag{3.27}$$

$$\sigma^2=\frac{3}{2}\Delta H^2, \tag{3.28}$$

where H_i ($i = 1, 2, 3$) represents the directional Hubble parameters in the direction x, y and z respectively.

4 Stiff Domain Walls ($\rho = p$)

Adding (3.3) and (3.4) we get

$$\frac{d}{dt}\left(\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)+\left(\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)\left(\frac{\dot{a}_1}{a_1}+\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)=0 \tag{4.1}$$

Now, from (3.9) and (4.1), we get

$$\frac{d}{dt}\left(\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)+\left(\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)(3H)=0$$

Integrating, which gives

$$\left(\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)=\beta_2 e^{-3kt}, \quad \text{for } m=0. \tag{4.2}$$

$$\left(\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)=\frac{\beta_2}{(mkt+c_2)^{\frac{3}{m}}}, \quad \text{for } m\neq 0 \text{ and } 3, \tag{4.3}$$

and

$$\left(\frac{\dot{a}_2}{a_2}+\frac{\dot{a}_3}{a_3}\right)=\frac{\beta_2}{(3kt+c_2)},\qquad \text{for } m=3, \tag{4.4}$$

where $\beta_2 =$ constant of integration.

4.1 Model for $m = 0$ (for $q = -1$)

Integrating (3.19) and (4.2) respectively gives

$$\frac{a_2}{a_3}=d_1\exp\left(-\frac{\beta_1}{3k}e^{-3kt}\right), \tag{4.1.1}$$

$$a_2a_3=d_2\exp\left(-\frac{\beta_2}{3k}e^{-3kt}\right), \tag{4.1.2}$$

where d_1 and d_2 are constants of integration.

From (4.1.1) and (4.1.2) we get the expressions for the scale factors as

$$a_2=\sqrt{d_1d_2}\exp\left(-\frac{(\beta_1+\beta_2)}{6k}e^{-3kt}\right), \tag{4.1.3}$$

$$a_3=\sqrt{\frac{d_1}{d_2}}\exp\left(\frac{\beta_1-\beta_2}{6k}e^{-3kt}\right). \tag{4.1.4}$$

Using (4.1.3) and (4.1.4) in (3.12) we get

$$a_1=d_3\exp\left(3kt+\frac{\beta_2}{3k}e^{-3kt}\right), \tag{4.1.5}$$

where $d_3 = c_1/d_2$ is constant of integration.

The spatial volume of the universe is found as

$$V=c_1e^{3kt} \tag{4.1.6}$$

The directional Hubble parameters as defined in (3.10) are found as

$$H_x=\frac{\dot{a}_1}{a_1}=3k-\beta_2e^{-3kt}, \tag{4.1.7}$$

$$H_y=\frac{\dot{a}_2}{a_2}=\left(\frac{\beta_2+\beta_1}{2}\right)e^{-3kt}, \tag{4.1.8}$$

$$H_z=\frac{\dot{a}_3}{a_3}=\left(\frac{\beta_2-\beta_1}{2}\right)e^{-3kt}. \tag{4.1.9}$$

By using (3.26) to (3.28) various physical parameters are found as

Expansion scalar:

$$\theta=3k. \tag{4.1.10}$$

Anisotropy parameter:

$$\Delta = \frac{1}{4k^2}[12k^2 - 4ke^{-3kt}(2\beta_2 + \beta_1) + (3\beta_2^2 + \beta_1^2)e^{-6kt}]. \quad (4.1.11)$$

Shear scalar:

$$\sigma^2 = \frac{3}{8}[12k^2 - 4ke^{-3kt}(2\beta_2 + \beta_1) + (3\beta_2^2 + \beta_1^2)e^{-6kt}]. \quad (4.1.12)$$

Using (3.12) and (3.24) we get the rest energy density of the domain walls as

$$\rho = \frac{\ddot{V}}{2V} = \frac{9k^2}{2} \quad (4.1.13)$$

Using (4.1.3), (4.1.4), (4.1.5) and (4.1.13) in (3.3) we get, creation field as

$$\begin{aligned}
\dot{C}^2 &= \frac{2}{f}\left\{\frac{9k^2}{2} - 3k\beta_2 e^{-3kt} + \left[\beta_2 - \left(\frac{\beta_2^2 - \beta_1^2}{4}\right)\right]e^{-6kt}\right\} \\
\dot{C} &= \sqrt{\frac{1}{f}}3k\left\{1 + \frac{2}{9k^2}\left[-3k\beta_2 e^{-3kt} + \left(\frac{4\beta_2 - \beta_2^2 + \beta_1^2}{4}\right)e^{-6kt}\right]\right\}^{1/2} \\
\dot{C} &\approx \sqrt{\frac{1}{f}}3k\left\{1 + \frac{1}{9k^2}\left[-3k\beta_2 e^{-3kt} + \left(\frac{4\beta_2 - \beta_2^2 + \beta_1^2}{4}\right)e^{-6kt}\right]\right\} \quad \text{(to the 1}^{\text{st}}\text{ order)} \\
\dot{C} &= \sqrt{\frac{1}{f}}\left\{3k + \frac{1}{3k}\left[-3k\beta_2 e^{-3kt} + \left(\frac{4\beta_2 - \beta_2^2 + \beta_1^2}{4}\right)e^{-6kt}\right]\right\} \\
C &= \sqrt{\frac{2}{f}}\left\{\frac{3kt}{2} + \frac{\beta_2 e^{-3kt}}{3k} - \left(\frac{4\beta_2 - \beta_2^2 + \beta_1^2}{72k^2}\right)e^{-6kt}\right\}.
\end{aligned} \quad (4.1.14)$$

4.1.1 Physical Behavior of the Model

For this model, we get $q = -1$ and $\frac{dH}{dt} = 0$. This implies that the value of Hubble parameter is greatest and the rate of expansion of the universe is also fastest. Therefore, this model represents the inflationary era in the early universe and the very late times of the universe.

The spatial volume V is finite at $t = 0$, expands exponentially as t increases and become infinitely large as $t \to \infty$ as shown in Fig. 1.

Here the energy density is $\rho = \frac{9k^2}{2} =$ constant (i.e. $\dot{\rho} = 0$) through out the evolution and hence there is no big bang type of singularity.

The expansion scalar is constant throughout the evolution of the universe.

Also our C-field start with finite value and becomes infinitely large as $t \to \infty$ as shown in Fig. 2. We can interpret our result as the matter is suppose to move along the geodesic normal to the surface $t =$ constant [42, 47]. As the matter moves further apart, it is assumed that more mass is continuously created to maintain the matter density.

The ratio $\frac{\sigma^2}{\theta^2} = \frac{1}{24k^2}[12k^2 - 4ke^{-3kt}(2\beta_2 + \beta_1) + (3\beta_2^2 + \beta_1^2)e^{-6kt}] \to \frac{1}{2}$ as $t \to \infty$. Hence the anisotropy is maintained through out.

Fig. 1 $C(t)$ vs. t
$[f=2, \beta_1=\beta_2=k=1]$

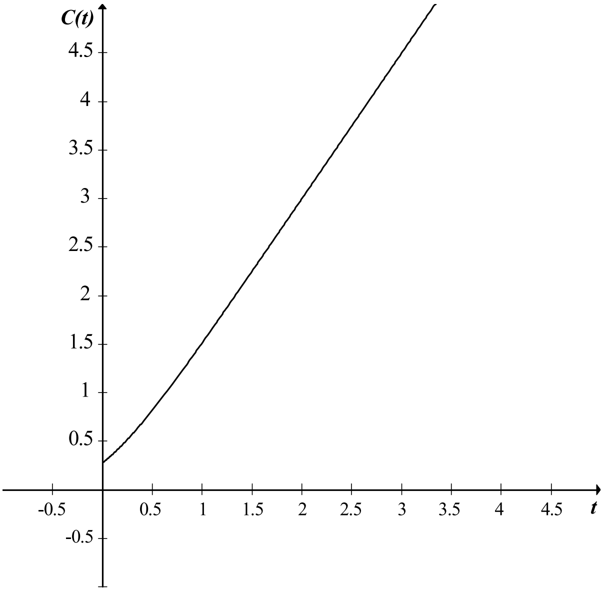
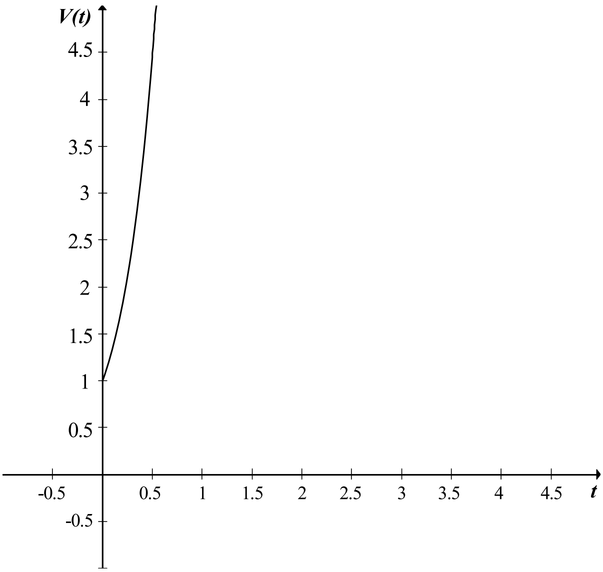


Fig. 2 Volumetric expansion
$[f=2, \beta_1=\beta_2=k=1]$



4.2 Model for $m \neq 0$ ($q \neq -1$)

In this subsection, the solutions are valid for all possible values of m except for $m=0$ and $m=3$.

Integrating (3.20) and (4.3) respectively, we get

$$\frac{a_2}{a_3} = d_1 \exp\left(\frac{\beta_1}{k(m-3)}(mkt+c_2)^{\frac{m-3}{m}}\right) \tag{4.2.1}$$

$$a_2 a_3 = d_2 \exp\left(\frac{\beta_2}{k(m-3)}(mkt+c_2)^{\frac{m-3}{m}}\right) \tag{4.2.2}$$

where d_1 and d_2 are constants of integration.

From (4.2.1) and (4.2.2) we get the expressions for the scale factors as

$$a_2 = \sqrt{d_1 d_2}\exp\left(\frac{(\beta_1+\beta_2)}{2k(m-3)}(mkt+c_2)^{\frac{m-3}{m}}\right), \tag{4.2.3}$$

$$a_3 = \sqrt{\frac{d_1}{d_2}}\exp\left(\frac{\beta_2-\beta_1}{2k(m-3)}(mkt+c_2)^{\frac{m-3}{m}}\right). \tag{4.2.4}$$

Using (4.2.3) and (4.2.4) in (3.13) we get

$$a_1 = (mkt+c_2)^{\frac{3}{m}}\exp\left(-\frac{\beta_2}{k(m-3)}(mkt+c_2)^{\frac{m-3}{m}}\right). \tag{4.2.5}$$

The spatial volume of the universe is found as

$$V = (mkt+c_2)^{\frac{3}{m}} \tag{4.2.6}$$

The directional Hubble parameters as defined in (3.10) are found as

$$H_x = \frac{\dot{a}_1}{a_1} = 3k(mkt+c_2)^{-1} - \beta_2(mkt+c_2)^{\frac{-3}{m}}, \tag{4.2.7}$$

$$H_y = \frac{\dot{a}_2}{a_2} = \left(\frac{\beta_2+\beta_1}{2}\right)(mkt+c_2)^{\frac{-3}{m}}, \tag{4.2.8}$$

$$H_z = \frac{\dot{a}_3}{a_3} = \left(\frac{\beta_2-\beta_1}{2}\right)(mkt+c_2)^{\frac{-3}{m}}. \tag{4.2.9}$$

By using (3.26) to (3.28), the various physical parameters are found as

Expansion scalar:

$$\theta = 3k(mkt+c_2)^{-1} \tag{4.2.10}$$

Anisotropy parameter:

$$\Delta = \frac{1}{4k^2(mkt+c_2)^{-2}}\left[\frac{12k^2}{(mkt+c_2)^2} - \frac{12k\beta_2}{(mkt+c_2)^{(\frac{m+3}{m})}} + \frac{(3\beta_2^2+2\beta_1^2)}{(mkt+c_2)^{6/m}}\right]. \tag{4.2.11}$$

Shear scalar:

$$\sigma^2 = \frac{3}{8}\left[\frac{12k^2}{(mkt+c_2)^2} - \frac{12k\beta_2}{(mkt+c_2)^{(\frac{m+3}{m})}} + \frac{(3\beta_2^2+2\beta_1^2)}{(mkt+c_2)^{6/m}}\right] \tag{4.2.12}$$

For stiff domain walls from (3.24) and using (3.12) we get

$$\rho = \frac{\ddot{V}}{2V} = \frac{3(3-m)k^2}{2(mkt+c_2)^2}. \tag{4.2.13}$$

Using (4.1.3), (4.1.4), (4.1.5) and (4.1.13) in equation (3.3) we get

$$\dot{C}^2 = \frac{2}{f}\left\{\frac{3(3-m)k^2}{2(mkt+c_2)^2} + (mkt+c_2)^{-6/m}(3\beta_2^2+\beta_1^2) - 3k\beta_2(mkt+c_2)^{\frac{-(m+3)}{m}}\right\}$$

$$\dot{C} = \sqrt{\frac{1}{f}}\frac{\sqrt{3(3-m)}k}{(mkt+c_2)} \times\left\{1+\frac{2(mkt+c_2)^2}{3(3-m)k^2}\left[(mkt+c_2)^{-6/m}(3\beta_2^2+\beta_1^2) - 3k\beta_2(mkt+c_2)^{\frac{-(m+3)}{m}}\right]\right\}^{1/2}$$

$$\dot{C} \approx \sqrt{\frac{1}{f}}\frac{\sqrt{3(3-m)}k}{(mkt+c_2)} \times\left\{1+\frac{(mkt+c_2)^2}{3(3-m)k^2}\left[(mkt+c_2)^{-6/m}(3\beta_2^2+\beta_1^2) - 3k\beta_2(mkt+c_2)^{\frac{-(m+3)}{m}}\right]\right\}$$

(to the 1st order) (4.2.14)

$$\dot{C} = \sqrt{\frac{3(3-m)}{f}}\left\{\frac{k}{(mkt+c_2)} + \frac{(mkt+c_2)}{3(3-m)k} \times\left[(mkt+c_2)^{-6/m}(3\beta_2^2+\beta_1^2) - 3k\beta_2(mkt+c_2)^{\frac{-(m+3)}{m}}\right]\right\}$$

$$C = \sqrt{\frac{3(3-m)}{f}} \times\left\{\frac{1}{m}\log(mkt+c_2) - \frac{(3\beta_2^2+\beta_1^2)}{3k^2(3-m)^2}(mkt+c_2)^{\frac{2(m-3)}{m}} + \frac{\beta_2}{k(3-m)^2}(mkt+c_2)^{\frac{(m-3)}{m}}\right\}.$$

4.2.1 Physical Behavior of the Model

For this model, we have $q<0$ for $0\le m\le 1$. Which indicates that the universe is accelerating. In particular for $m=1$, we get $q=0$ indicating that the universe is expanding with constant velocity. Figure 4 shows that the spatial volume V is finite at $t=0$ and become infinitely large as $t\to\infty$.

Moreover as $t\to 0$, $\rho=\frac{3(3-m)k^2}{2c_2^2}=$ constant (i.e., $\dot{\rho}\to 0$) and as $t\to\infty$, $\rho\to 0$ as in Fig. 5. Hence, there is no big bang type of singularity. Here we obtain steady state at $t=0$ and the model reduces to vacuum as $t\to\infty$.

Also our C-field start with finite value and becomes infinitely large as $t\to\infty$ as shown in Fig. 3.

The ratio

$$\frac{\sigma^2}{\theta^2} = \frac{3}{24k^2}\left[12k^2 - 12k\beta_2(mkt+c_2)^{(\frac{m-3}{m})} + (3\beta_2^2+2\beta_1^2)(mkt+c_2)^{2(\frac{m-3}{m})}\right] \to \frac{3}{2}$$

as $t\to\infty$ only for $0<m<3$. Hence the anisotropy is maintained throughout.

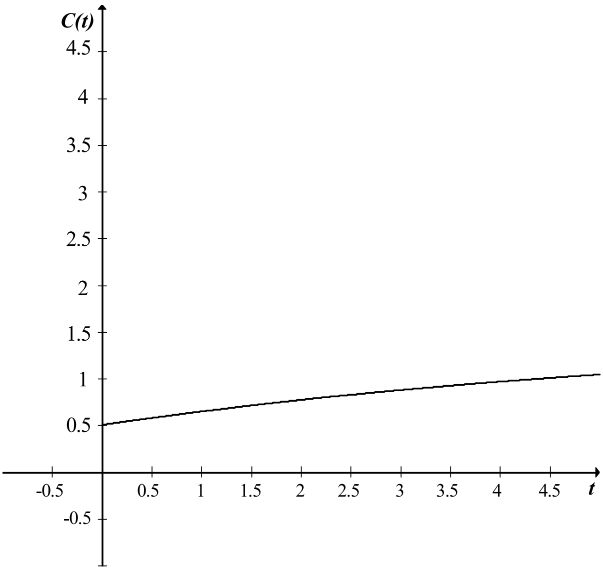


Fig. 3 $C(t)$ vs. t $(k=1, m=1, c_2=2)$

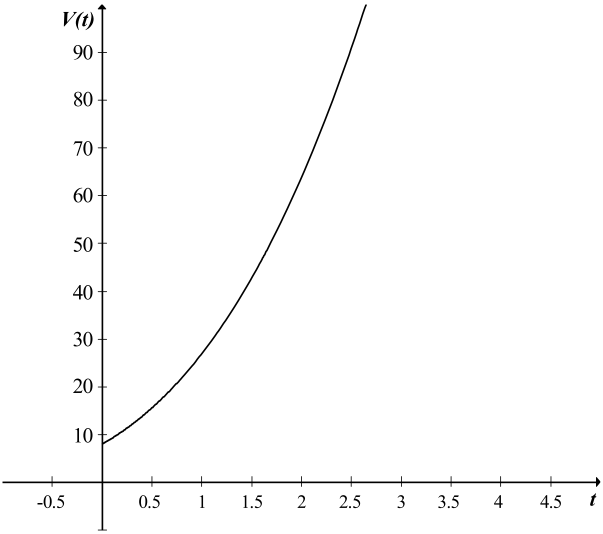


Fig. 4 Volumetric expansion $(k=1, m=1, c_2=2)$

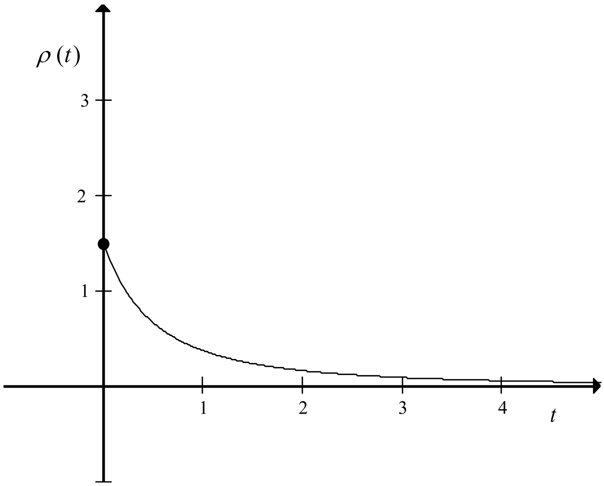
4.3 Model for $m=3$ $(q=2)$

In this case we arrive at null matter energy density which indicates empty universe. Hence, this case is not interesting to discuss.

5 Conclusion

In this paper, for $m=0$ $(q=-1)$, we get the steady state cosmological model of the universe. We can interpret this result as the matter is supposed to move along the geodesic

Fig. 5 Evolution of density ($k = 1, m = 1, c_2 = 1$)



normal to the surface $t =$ constant. As the matter moves further apart, it is assumed that more mass is continuously created to maintain the matter density its constant value [42, 47]. Also in this case, we get, negative deceleration parameter indicating that the universe is accelerating which is consistent with the present day observations. Perlmutter et al. [48, 49] and Riess et al. [50] have shown that the decelerating parameter of the universe is in the range $-1 \leq q \leq 0$ and the present day universe is undergoing accelerated expansion. For $m \neq 0, 3$ ($q \neq -1$) the universe starts with constant density and reduces to vacuum for late times. There is no big bang type of singularity in both the models and their anisotropy is maintained throughout.

Here, an attempt has been made to introduce domain walls with creation field to study the space-time geometry corresponding to Bianchi type-I universe. In this work, we have shown that domain walls are consistent in C-field theory with constant deceleration parameter. The solutions obtained here are important as they are perhaps the first analytical solutions for domain walls in presence of C-field.

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